Nađimo prvo funkciju sistema H(z). Stavimo li $x_n = z^n$, tada je $y_n = z^n\,\mathrm{H}(z)$, tako da diferentna jednačina postaje $3\,z^n\,\mathrm{H}(z) - z^{n-1}\,\mathrm{H}(z) = z^n + z^{n-1}$. Odavde slijedi:

$$\mathrm{H}(z) = \frac{z^n + z^{n-1}}{3\,z^n - z^{n-1}} = \frac{z+1}{3\,z-1}$$

Nađimo sada z-transformaciju X(z) pobude $x_n = n\,2^n$ za $n \geq 0$ i $x_n = 0$ za $n < 0$. Prema osobinama z-transformacije i tablicama imamo:

$$\mathcal{Z}\{n\,2^n\} = -z\,\frac{\mathrm{d}}{\mathrm{d}z}\left[\mathcal{Z}\{2^n\}\right] = -z\,\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{z}{z-2}\right] = -z\,\frac{-2}{(z-2)^2} = \frac{2\,z}{(z-2)^2}$$

Odziv y_n na pobudu $x_n = n\,2^n$ za $n \geq 0$ i $x_n = 0$ sada možemo naći primjenom teoreme o konvoluciji:

$$y_n = \mathcal{Z}^{-1}\{n\,2^n\,\mathrm{H}(z)\} = \mathcal{Z}^{-1}\left\{\frac{2\,z\,(z+1)}{(z-2)^2\,(3\,z-1)}\right\}$$

Primijenimo sada postupak za nalaženje inverzne z-transformacije racionalne funkcije. Rastava $\mathrm{Y}(z) = z\,\mathrm{P}(z)\,/\,\mathrm{Q}(z)$ sada daje $\mathrm{P}(z) = 2\,(z+1)$, $\mathrm{Q}(z) = (z-2)^2(3\,z-1) = 3\,(z-2)^2(z-1/3)$, $z_1 = 2$, $z_2 = 1/3$, $v_2 = 1$, $\mathrm{Q}_1(z) = 3\,(z-1/3) = 3\,z-1$ i $\mathrm{Q}_2(z) = 3\,(z-2)^2$, tako da formula za inverznu z-transformaciju racionalne funkcije daje

$$y_n = \sum_{i=0}^{1} \frac{1}{(1-i)!}\frac{\mathrm{d}^{1-i}}{\mathrm{d}z^{1-i}}\left[\frac{\mathrm{P}(z)}{\mathrm{Q}_1(z)}\right]_{z=2}\binom{n}{i}\,2^{n-i} + \frac{\mathrm{P}(1/3)}{\mathrm{Q}_2(1/3)}\left(\frac{1}{3}\right)^n =$$

$$= \sum_{i=0}^{1} \frac{1}{(1-i)!}\frac{\mathrm{d}^{1-i}}{\mathrm{d}z^{1-i}}\left[\frac{2\,(z+1)}{3\,z-1}\right]_{z=2}\binom{n}{i}\,2^{n-i} + \frac{8}{25}\left(\frac{1}{3}\right)^n =$$

$$= \frac{1}{1!}\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{2\,(z+1)}{3\,z-1}\right]_{z=2} 2^n + \frac{1}{0!}\left[\frac{2\,(z+1)}{3\,z-1}\right]_{z=2} n\,2^{n-1} + \frac{8}{25}\left(\frac{1}{3}\right)^n =$$

$$= \left[\frac{-8}{(3\,z-1)^2}\right]_{z=2} 2^n + \frac{6}{5}\,n\,2^{n-1} + \frac{8}{25}\left(\frac{1}{3}\right)^n = \frac{15\,n-8}{5}\,2^n + \frac{8}{25}\left(\frac{1}{3}\right)^n,\; n \geq 0$$

Što se tiče odziva na pobudu $x_n = \cos(n\pi/4)$, $n \in \mathbb{Z}$, kako je pobuda nekauzalna, iskorističemo činjenice da odziv na pobudu oblika z^n glasi $z^n\,\mathrm{H}(z)$, da je $\cos(n\pi/4) = (e^{i\,n\pi/4} + e^{-i\,n\pi/4})\,/\,2$, kao i da je sistem linearan. Stoga je:

$$y_n = \frac{1}{2}\left(e^{n\pi i/4}\mathrm{H}(e^{\pi i/4}) + e^{-n\pi i/4}\mathrm{H}(e^{-\pi i/4})\right) = \mathrm{Re}\left(e^{n\pi i/4}\mathrm{H}(e^{\pi i/4})\right) =$$

$$= \mathrm{Re}\left((\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4})\frac{e^{\pi i/4}+1}{3e^{\pi i/4}-1}\right) = \mathrm{Re}\left((\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4})\frac{2+\sqrt{2}+i\sqrt{2}}{-2+3\sqrt{2}+3i\sqrt{2}}\right) =$$

$$= \mathrm{Re}\left((\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4})\frac{2+\sqrt{2}+i\sqrt{2}}{-2+3\sqrt{2}+3i\sqrt{2}}\,\frac{-2+3\sqrt{2}-3i\sqrt{2}}{-2+3\sqrt{2}-3i\sqrt{2}}\right) =$$

$$= \mathrm{Re}\left((\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4})\frac{2+\sqrt{2}-2i\sqrt{2}}{10-3\sqrt{2}}\right) = \frac{2+\sqrt{2}}{10-3\sqrt{2}}\cos\frac{n\pi}{4} + \frac{2\sqrt{2}}{10-3\sqrt{2}}\sin\frac{n\pi}{4} =$$

$$= \frac{2+\sqrt{2}}{10-3\sqrt{2}}\,\frac{10+3\sqrt{2}}{10+3\sqrt{2}}\cos\frac{n\pi}{4} + \frac{2\sqrt{2}}{10-3\sqrt{2}}\,\frac{10+3\sqrt{2}}{10+3\sqrt{2}}\sin\frac{n\pi}{4} =$$

$$= \frac{13+8\sqrt{2}}{41}\cos\frac{n\pi}{4} + \frac{6+10\sqrt{2}}{41}\sin\frac{n\pi}{4},\; n \in \mathbb{Z}$$