Prvo ćemo odrediti funkciju sistema H(z). Stavimo li $x_n = z^n$, tada je $y_n = z^n\,\mathrm{H}(z)$, pa diferentna jednačina dobija oblik

$$(z^n + 5\,z^{n-1} + 6\,z^{n-2})\,\mathrm{H}(z) = z^n$$

Odavde je:

$$\mathrm{H}(z) = \frac{z^n}{z^n + 5\,z^{n-1} + 6\,z^{n-2}} = \frac{z^2}{z^2 + 5\,z + 6} = \frac{z^2}{(z+2)\,(z+3)}$$

Impulsni odziv h_n možemo naći kao inverznu z-transformaciju od H(z), tj. kao $h_n = \mathcal{Z}^{-1}\{\mathrm{H}(z)\}$. Rastava $\mathrm{H}(z) = z\,\mathrm{P}(z)\,/\,\mathrm{Q}(z)$ daje:

$$\mathrm{P}(z) = z, \qquad \mathrm{Q}(z) = (z+2)\,(z+3)$$

Polinom Q(z) ima dvije proste nule $z_1 = -2$ i $z_2 = -3$. Dalje je:

$$\mathrm{Q}_1(z) = \mathrm{Q}(z)\,/\,(z - z_1) = z + 3$$
$$\mathrm{Q}_2(z) = \mathrm{Q}(z)\,/\,(z - z_2) = z + 2$$

Sada, na osnovu formule za inverznu z-transformaciju racionalne funkcije, dobijamo:

$$h_n = \frac{\mathrm{P}(-2)}{\mathrm{Q}_1(-2)}(-2)^n + \frac{\mathrm{P}(-3)}{\mathrm{Q}_1(-3)}(-3)^n = \frac{-2}{-2+3}(-2)^n + \frac{-3}{-3+2}(-3)^n =$$
$$= -2\,(-2)^n + 3\,(-3)^n = (-2)^{n+1} - (-3)^{n+1}$$

Za rješavanje drugog dijela zadatka, poslužićemo se činjenicom da se odziv y_n na proizvoljnu kauzalnu pobudu x_n može dobiti kao $y_n = \mathcal{Z}^{-1}\{\mathrm{Y}(z)\}$ gdje je $\mathrm{Y}(z) = \mathrm{X}(z)\,\mathrm{H}(z)$ i $\mathrm{X}(z) = \mathcal{Z}\{x_n\}$. Z-transformaciju pobude x_n direktno očitavamo iz tablica:

$$\mathrm{X}(z) = \mathcal{Z}\{x_n\} = \mathcal{Z}\{2^n \sin\frac{n\pi}{3}\} = \frac{2\,z\sin\frac{\pi}{3}}{z^2 - 2\cdot 2\,z\cos\frac{\pi}{3} + 2^2} = \frac{\sqrt{3}\,z}{z^2 - 2\,z + 4}$$

Stoga je:

$$\mathrm{Y}(z) = \mathrm{X}(z)\,\mathrm{H}(z) = \frac{\sqrt{3}\,z^3}{(z+2)\,(z+3)\,(z^2 - 2\,z + 4)}$$

Sada, rastava $\mathrm{Y}(z) = z\,\mathrm{P}(z)\,/\,\mathrm{Q}(z)$ daje:

$$\mathrm{P}(z) = \sqrt{3}\,z^2, \qquad \mathrm{Q}(z) = (z+2)\,(z+3)\,(z^2 - 2\,z + 4)$$

Ovaj polinom Q(z) ima četiri proste nule

$$z_1 = -2; \qquad z_2 = -3; \qquad z_3 = 1 + i\sqrt{3}\,; \qquad z_4 = 1 - i\sqrt{3}\,;$$

Dalje imamo:

$$\mathrm{Q}_1(z) = \mathrm{Q}(z)\,/\,(z - z_1) = (z+3)\,(z^2 - 2\,z + 4)$$
$$\mathrm{Q}_2(z) = \mathrm{Q}(z)\,/\,(z - z_2) = (z+2)\,(z^2 - 2\,z + 4)$$
$$\mathrm{Q}_3(z) = \mathrm{Q}(z)\,/\,(z - z_3) = (z^2 + 5\,z + 6)\,(z - 1 + i\sqrt{3}\,)$$
$$\mathrm{Q}_4(z) = \mathrm{Q}(z)\,/\,(z - z_4) = (z^2 + 5\,z + 6)\,(z - 1 - i\sqrt{3}\,)$$

Primjena formule za inverznu z-transformaciju racionalne funkcije sada daje:

$$y_n = \frac{P(-2)}{Q_1(-2)}(-2)^n + \frac{P(-3)}{Q_2(-3)}(-3)^n + \frac{P(1+i\sqrt{3})}{Q_3(1+i\sqrt{3})}(1+i\sqrt{3})^n + \frac{P(1-i\sqrt{3})}{Q_4(1-i\sqrt{3})}(1-i\sqrt{3})^n$$

Ovdje je korisno prvo posebno izračunati pomoćne veličine koje se javljaju u ovom izrazu:

$$P(-2) = \sqrt{3}\cdot(-2)^2 = 4\sqrt{3}$$
$$P(-3) = \sqrt{3}\cdot(-3)^2 = 9\sqrt{3}$$
$$P(1+i\sqrt{3}) = \sqrt{3}\cdot(1+i\sqrt{3})^2 = \sqrt{3}\cdot(1+2i\sqrt{3}-3) = \sqrt{3}\cdot(-2+2i\sqrt{3}) = -2\sqrt{3}+6i$$
$$P(1-i\sqrt{3}) = \sqrt{3}\cdot(1-i\sqrt{3})^2 = \sqrt{3}\cdot(1-2i\sqrt{3}-3) = \sqrt{3}\cdot(-2-2i\sqrt{3}) = -2\sqrt{3}-6i$$

$$Q_1(-2) = (-2+3)\,((-2)^2-2\,(-2)+4) = 12$$
$$Q_2(-3) = (-3+2)\,((-3)^2-2\,(-3)+4) = -10$$
$$Q_3(1+i\sqrt{3}) = ((1+i\sqrt{3})^2+5(1+i\sqrt{3})+6)(1+i\sqrt{3}-1+i\sqrt{3}) = (1+2i\sqrt{3}-3+5+5i\sqrt{3}+6)\cdot 2i\sqrt{3} =$$
$$= (9+7i\sqrt{3})\cdot 2i\sqrt{3} = -42+18i\sqrt{3}$$
$$Q_4(1-i\sqrt{3}) = ((1-i\sqrt{3})^2+5(1-i\sqrt{3})+6)(1-i\sqrt{3}-1-i\sqrt{3}) = (1-2i\sqrt{3}-3+5-5i\sqrt{3}+6)\cdot(-2i\sqrt{3}) =$$
$$= (9-7i\sqrt{3})\cdot(-2i\sqrt{3}) = -42-18i\sqrt{3}$$

Zapravo, odmah se zna da je $Q_4(1-i\sqrt{3})$ konjugovano kompleksna vrijednost od $Q_3(1+i\sqrt{3})$ i bez potrebe da se to posebno računa. Konačno je:

$$y_n = \frac{4\sqrt{3}}{12}(-2)^n + \frac{9\sqrt{3}}{-19}(-3)^n + \frac{-2\sqrt{3}+6i}{-42+18i\sqrt{3}}(1+i\sqrt{3})^n + \frac{-2\sqrt{3}-6i}{-42-18i\sqrt{3}}(1-i\sqrt{3})^n =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + \frac{-\sqrt{3}+3i}{-21+9i\sqrt{3}}(1+i\sqrt{3})^n + \frac{-\sqrt{3}-3i}{-21-9i\sqrt{3}}(1-i\sqrt{3})^n =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + \frac{(-\sqrt{3}+3i)(-21-9i\sqrt{3})}{(-21+9i\sqrt{3})(-21-9i\sqrt{3})}(1+i\sqrt{3})^n + \frac{(-\sqrt{3}-3i)(-21+9i\sqrt{3})}{(-21-9i\sqrt{3})(-21+9i\sqrt{3})}(1-i\sqrt{3})^n =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + \frac{48\sqrt{3}-36i}{21^2+3\cdot 9^2}(1+i\sqrt{3})^n + \frac{48\sqrt{3}+36i}{21^2+3\cdot 9^2}(1-i\sqrt{3})^n =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + \frac{48\sqrt{3}-36i}{684}(1+i\sqrt{3})^n + \frac{48\sqrt{3}+36i}{684}(1-i\sqrt{3})^n =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + \left(\frac{4\sqrt{3}}{57} - \frac{1}{19}i\right)(1+i\sqrt{3})^n + \left(\frac{4\sqrt{3}}{57} + \frac{1}{19}i\right)(1-i\sqrt{3})^n =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + \left(\frac{4\sqrt{3}}{57} - \frac{1}{19}i\right)2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right) + \left(\frac{4\sqrt{3}}{57} + \frac{1}{19}i\right)2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right) =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + \frac{8\sqrt{3}}{57}2^n\cos\frac{n\pi}{3} + \frac{2}{19}2^n\sin\frac{n\pi}{3} =$$
$$= \frac{\sqrt{3}}{3}(-2)^n - \frac{9\sqrt{3}}{19}(-3)^n + 2^n\left(\frac{8\sqrt{3}}{57}\cos\frac{n\pi}{3} + \frac{2}{19}\sin\frac{n\pi}{3}\right)$$