a) Prvo ćemo odrediti funkciju sistema H(z). Ako uzmemo da je $x_n = z^n$, tada je $y_n = z^n$ H(z), pa imamo:

$$z^n \mathrm{H}(z) - z^{n-1} \mathrm{H}(z) - 6\, z^{n-2} \mathrm{H}(z) = z^{n-1} + 2\, z^{n-3}$$

Odavde je:

$$\mathrm{H}(z) = \frac{z^{n-1} + 2\, z^{n-3}}{z^n - z^{n-1} - 6\, z^{n-2}} = \frac{z^{-1} + 2\, z^{-3}}{1 - z^{-1} - 6\, z^{-2}} = \frac{z^2 + 2}{z^3 - z^2 - 6\, z} = \frac{z^2 + 2}{z\,(z+2)\,(z-3)}$$

Impulsni odziv h_n ćemo naći kao inverznu z-transformaciju od H(z), tj. $h_n = \mathcal{Z}^{-1}\{\mathrm{H}(z)\}$. Rastava H($z$) = z P(z) / Q(z) daje:

$$\mathrm{P}(z) = z^2 + 2, \qquad \mathrm{Q}(z) = z^2\,(z+2)\,(z-3)$$

Nule polinoma Q(z) sa pripadnim višestrukostima su:

$$z_1 = 0,\ \nu_1 = 2; \qquad z_2 = -2,\ \nu_2 = 1; \qquad z_3 = 3,\ \nu_3 = 1;$$

Dalje imamo:

$$\mathrm{Q}_1(z) = \mathrm{Q}(z) / (z - z_1)^2 = (z+2)\,(z-3) = z^2 - z - 6$$
$$\mathrm{Q}_2(z) = \mathrm{Q}(z) / (z - z_2) = z^2\,(z-3)$$
$$\mathrm{Q}_3(z) = \mathrm{Q}(z) / (z - z_3) = z^2\,(z+2)$$

Sada možemo koristiti opću formulu za inverznu z-transformaciju racionalne funkcije. Član koji sadrži izvod rastavićemo po Leibinzovoj formuli, pri čemu se faktore oblika 0^{n-k} treba zamijeniti sa δ_{n-k}:

$$x_n = \sum_{i=0}^{1} \frac{1}{(1-i)!} \frac{\mathrm{d}^{1-i}}{\mathrm{d}z^{1-i}} \left[\frac{\mathrm{P}(z)}{\mathrm{Q}_1(z)}\right]_{z=0} \binom{n}{i} \delta_{n-i} + \frac{\mathrm{P}(-2)}{\mathrm{Q}_2(-2)} (-2)^n + \frac{\mathrm{P}(3)}{\mathrm{Q}_3(3)} 3^n =$$
$$= \frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{z^2+2}{z^2 - z - 6}\right]_{z=0} \binom{n}{0} \delta_n + \left[\frac{z^2+2}{z^2 - z - 6}\right]_{z=0} \binom{n}{1} \delta_{n-1} - \frac{6}{20} (-2)^n + \frac{11}{45} 3^n =$$
$$= \left[\frac{2\,z\,(z^2 - z - 6) - (z^2+2)\,(2\,z - 1)}{(z^2 - z - 6)^2}\right]_{z=0} \delta_n - \frac{2}{6}\, n\, \delta_{n-1} - \frac{3}{10} (-2)^n + \frac{11}{45} 3^n =$$
$$= \frac{1}{18} \delta_n - \frac{1}{3} \delta_{n-1} - \frac{3}{10} (-2)^n + \frac{11}{45} 3^n \quad (\text{za } n \geq 0)$$

U posljednjem koraku je iskorištena činjenica da je $n\,\delta_{n-1} = \delta_{n-1}$.

b) Koristićemo činjenicu da je $y_n = \mathcal{Z}^{-1}\{\mathrm{Y}(z)\}$, gdje je Y($z$) = X($z$) H($z$) i X($z$) = $\mathcal{Z}\{x_n\}$. Na osnovu osobina z-transformacije i tablica imamo:

$$\mathrm{X}(z) = \mathcal{Z}\{n+2\} = \mathcal{Z}\{n\} + 2\, \mathcal{Z}\{1\} = \frac{z}{(z-1)^2} + 2\frac{z}{z-1} = \frac{z\,(2\,z - 1)}{(z-1)^2}$$

Stoga je:

$$\mathrm{Y}(z) = \mathrm{X}(z)\,\mathrm{H}(z) = \frac{(2\,z-1)\,(z^2+2)}{(z-1)^2\,(z+2)\,(z-3)}$$

Nastavljamo kao i u prethodnom slučaju. Rastava Y(z) = z P(z) / Q(z) daje:

$$\mathrm{P}(z) = (2z-1)(z^2+2) = 2z^3 - z^2 + 4z - 2, \qquad \mathrm{Q}(z) = z(z-1)^2(z+2)(z-3)$$

Nule polinoma Q(z) sa pripadnim višestrukostima su:

$$z_1 = 0, \nu_1 = 1; \quad z_2 = 1, \nu_2 = 2; \quad z_3 = -2, \nu_3 = 1; \quad z_4 = 3, \nu_3 = 1;$$

Odavde imamo:

$$\mathrm{Q}_1(z) = \mathrm{Q}(z)/(z-z_1) = (z-1)^2(z+2)(z-3)$$
$$\mathrm{Q}_2(z) = \mathrm{Q}(z)/(z-z_2)^2 = z(z+2)(z-3) = z^3 - z^2 - 6z$$
$$\mathrm{Q}_3(z) = \mathrm{Q}(z)/(z-z_3) = z(z-1)^2(z-3)$$
$$\mathrm{Q}_4(z) = \mathrm{Q}(z)/(z-z_3) = z(z-1)^2(z+2)$$

Stoga, na osnovu formule za inverznu z-transformaciju racionalne funkcije, imamo:

$$y_n = \frac{\mathrm{P}(0)}{\mathrm{Q}_1(0)}\delta_n + \sum_{i=0}^{1}\frac{1}{(1-i)!}\frac{\mathrm{d}^{1-i}}{\mathrm{dz}^{1-i}}\left[\frac{\mathrm{P}(z)}{\mathrm{Q}_2(z)}\right]_{z=1}\binom{n}{i}1^{n-i} + \frac{\mathrm{P}(-2)}{\mathrm{Q}_3(-2)}(-2)^n + \frac{\mathrm{P}(3)}{\mathrm{Q}_4(3)}3^n =$$

$$= \frac{2}{6}\delta_n + \frac{\mathrm{d}}{\mathrm{dz}}\left[\frac{2z^3 - z^2 + 4z - 2}{z^3 - z^2 - 6z}\right]_{z=1}\binom{n}{0} + \left[\frac{2z^3 - z^2 + 4z - 2}{z^3 - z^2 - 6z}\right]_{z=1}\binom{n}{1} - \frac{30}{90}(-2)^n + \frac{55}{60}3^n =$$

$$= \frac{1}{3}\delta_n + \left[\frac{(6z^2 - 2z + 4)(z^3 - z^2 - 6z) - (2z^3 - z^2 + 4z - 2)(3z^2 - 2z - 6)}{(z^3 - z^2 - 6z)^2}\right]_{z=1} - \frac{3}{6}n - \frac{1}{3}(-2)^n + \frac{11}{12}3^n =$$

$$= \frac{1}{3}\delta_n + \frac{11}{12} - \frac{1}{2}n - \frac{1}{3}(-2)^n + \frac{11}{12}3^n \quad (\text{za } n \geq 0)$$