a)

$$(\Delta x)_n = x_{n+1} - x_n = (n+1)^2\, 2^{n+1} - n^2\, 2^n = (n^2 + 2\,n + 1)\, 2^{n+1} - n^2\, 2^n =$$
$$= (2\,n^2 + 4\,n + 2)\, 2^n - n^2\, 2^n = (n^2 + 4\,n + 2)\, 2^n$$

$$(\nabla x)_n = x_n - x_{n-1} = n^2\, 2^n - (n-1)^2\, 2^{n-1} = n^2\, 2^n - (n^2 - 2\,n + 1)\, 2^{n-1} =$$
$$= 2\,n^2\, 2^{n-1} - (n^2 - 2\,n + 1)\, 2^{n-1} = (n^2 + 2\,n - 1)\, 2^{n-1}$$

b)

$$(\Delta x)_n = x_{n+1} - x_n = \sin \Omega\,(n+1) - \sin \Omega\, n = 2 \cos \frac{\Omega(n+1) + \Omega\, n}{2} \sin \frac{\Omega(n+1) - \Omega\, n}{2} =$$
$$= 2 \sin \frac{\Omega}{2} \cos (\Omega\, n + \frac{\Omega}{2})$$

$$(\nabla x)_n = x_n - x_{n-1} = \sin \Omega - \sin \Omega\,(n-1) = 2 \cos \frac{\Omega\, n + \Omega(n-1)}{2} \sin \frac{\Omega\, n - \Omega(n-1)}{2} =$$
$$= 2 \sin \frac{\Omega}{2} \cos (\Omega\, n - \frac{\Omega}{2})$$

c)

$$(\Delta x)_n = x_{n+1} - x_n = \ln\,(a\,(n+1)) - \ln\,(a\,n) = \ln \frac{n+1}{n} = \ln\,(1 + \frac{1}{n}),\ \ n > 0$$

$$(\Delta x)_n = x_n - x_{n-1} = \ln\,(a\,n) - \ln\,(a\,(n-1)) = \ln \frac{n}{n-1} = -\ln \frac{n-1}{n} = -\ln\,(1 - \frac{1}{n}),\ \ n > 1$$

d)

$$(\Delta x)_n = x_{n+1} - x_n = \frac{n+1}{(n+1)^2+1} - \frac{n}{n^2+1} = \frac{n+1}{n^2+2\,n+2} - \frac{n}{n^2+1} = \frac{(n+1)\,(n^2+1) - n\,(n^2+2\,n+2)}{(n^2+2\,n+2)\,(n^2+1)} =$$
$$= \frac{(n^3+n^2+n+1) - (n^3+2\,n^2+2\,n)}{(n^2+2\,n+2)\,(n^2+1)} = \frac{-n^2-n+1}{(n^2+2\,n+2)\,(n^2+1)}$$

$$(\nabla x)_n = x_n - x_{n-1} = \frac{n}{n^2+1} - \frac{n-1}{(n-1)^2+1} = \frac{n}{n^2+1} - \frac{n-1}{n^2-2\,n+2} = \frac{n\,(n^2-2\,n+2) - (n-1)\,(n^2+1)}{(n^2-2\,n+2)\,(n^2+1)} =$$
$$= \frac{(n^3-2\,n^2+2\,n) - (n^3-n^2+n-1)}{(n^2-2\,n+2)\,(n^2+1)} = \frac{-n^2+n+1}{(n^2-2\,n+2)\,(n^2+1)}$$