

Neka je  $A_i, i = 1..4$  događaj “ $i$ -ti lažov nije slagao”. Prema uvjetima zadatka je

$$p(A_1) = p(A_2) = p(A_3) = p(A_4) = \frac{1}{3}$$

Neka je  $B_i, i = 1..4$  događaj “ $i$ -ti lažov je rekao tačnu informaciju”. Prema postavci zadatka, veza između događaja  $B_i, i = 1..4$  i  $A_i, i = 1..4$  data je sljedećim relacijama:

$$\begin{aligned} B_1 &= A_1 \\ B_2 &= B_1 A_2 + \bar{B}_1 \bar{A}_2 \\ B_3 &= B_2 A_3 + \bar{B}_2 \bar{A}_3 \\ B_4 &= B_3 A_4 + \bar{B}_3 \bar{A}_4 \end{aligned}$$

Kako je ponašanje svakog lažova neovisno od ponašanja ostalih lažova, imamo:

$$\begin{aligned} p(B_1) &= p(A_1) = \frac{1}{3} \\ p(B_2) &= p(B_1) p(A_2) + p(\bar{B}_1) p(\bar{A}_2) = p(B_1) p(A_2) + (1 - p(B_1)) (1 - p(A_2)) = \\ &= \frac{1}{3} \cdot \frac{1}{3} + (1 - \frac{1}{3}) \cdot (1 - \frac{1}{3}) = \frac{5}{9} \\ p(B_3) &= p(B_2) p(A_3) + p(\bar{B}_2) p(\bar{A}_3) = p(B_2) p(A_3) + (1 - p(B_2)) (1 - p(A_3)) = \\ &= \frac{5}{9} \cdot \frac{1}{3} + (1 - \frac{5}{9}) \cdot (1 - \frac{1}{3}) = \frac{13}{27} \\ p(B_4) &= p(B_3) p(A_4) + p(\bar{B}_3) p(\bar{A}_4) = p(B_3) p(A_4) + (1 - p(B_3)) (1 - p(A_4)) = \\ &= \frac{13}{27} \cdot \frac{1}{3} + (1 - \frac{13}{27}) \cdot (1 - \frac{1}{3}) = \frac{41}{81} \end{aligned}$$

Mi zapravo tražimo vjerovatnoću  $p(B_1/B_4)$ , koja po Bayesovoj teoremi iznosi

$$p(B_1/B_4) = p(B_1) p(B_4/B_1) / p(B_4)$$

Nedostaje nam još vjerovatnoća  $p(B_4/B_1)$ , koju ćemo izračunati ovako:

$$\begin{aligned} p(B_2/B_1) &= p(A_2) = 1/3 \\ p(B_3/B_1) &= p(B_2/B_1) p(A_3) + p(\bar{B}_2/B_1) p(\bar{A}_3) = p(B_2/B_1) p(A_3) + (1 - p(B_2/B_1)) (1 - p(A_3)) = \\ &= \frac{1}{3} \cdot \frac{1}{3} + (1 - \frac{1}{3}) \cdot (1 - \frac{1}{3}) = \frac{5}{9} \\ p(B_4/B_1) &= p(B_3/B_1) p(A_4) + p(\bar{B}_3/B_1) p(\bar{A}_4) = p(B_3/B_1) p(A_4) + (1 - p(B_3/B_1)) (1 - p(A_4)) = \\ &= \frac{5}{9} \cdot \frac{1}{3} + (1 - \frac{5}{9}) \cdot (1 - \frac{1}{3}) = \frac{13}{27} \end{aligned}$$

Konačno je:

$$p(B_1/B_4) = (\frac{1}{3} \cdot \frac{13}{27}) / \frac{41}{81} = \frac{13}{41} \approx 31.7\%$$