

$$a) \quad \overline{x \vee y} = 1 - (1 - (1 - x)(1 - y)) = (1 - x)(1 - y) = 1 - x - y + xy$$

$$b) \quad \begin{aligned} x \vee (y \oplus z) &= x \vee y \bar{z} \vee \bar{y} z = 1 - (1 - x)(1 - y(1 - z))(1 - (1 - y)z) = \\ &= 1 - (1 - x)(1 - y + yz)(1 - z + yz) = 1 - (1 - x)(1 - z + yz - y + yz - y^2 z + yz - yz^2 + y^2 z^2) = \\ &= 1 - (1 - x)(1 - z + yz - y + yz - yz + yz - yz + yz) = 1 - (1 - x)(1 - y - z + 2yz) = \\ &= 1 - (1 - y - z + 2yz - x + xy + xz - 2xyz) = x + y + z - xy - xz - 2yz + 2xyz \end{aligned}$$

$$c) \quad \begin{aligned} x \vee \bar{y} z \vee \bar{x} y \bar{z} &= 1 - (1 - x)(1 - (1 - y)z)(1 - (1 - x)y(1 - z)) = \\ &= 1 - (1 - x)(1 - z + yz)(1 - y + xy + yz - xyz) = \\ &= 1 - (1 - x)(1 - y + xy + yz - xyz - z + yz - xyz - yz^2 + xy^2 z^2 + yz - y^2 z + xy^2 z + y^2 z^2 - xy^2 z^2) = \\ &= 1 - (1 - x)(1 - y + xy + yz - xyz - z + yz - xyz - yz + xyz + yz - yz + xyz + yz - xyz) = \\ &= 1 - (1 - x)(1 - y - z + xy + 2yz - xyz) = \\ &= 1 - 1 + y + z - xy - 2yz + xyz + x - xy - xz + x^2 y + 2xyz - x^2 yz = \\ &= y + z - xy - 2yz + xyz + x - xy - xz + xy + 2xyz - xyz = x + y + z - xy - xz - 2yz + 2xyz \end{aligned}$$

Vidi se da je dobijen isti rezultat kao pod b). Ovo je posljedica činjenice da su polazni izrazi pod c) i b) zapravo ekvivalentni. Zaista, imamo:

$$x \vee \bar{y} z \vee \bar{x} y \bar{z} = (x \vee \bar{x} y \bar{z}) \vee \bar{y} z = x \vee y \bar{z} \vee \bar{y} z = x \vee (y \oplus z)$$

$$d) \quad \begin{aligned} \overline{xy \vee yz \vee xz} &= 1 - (1 - (1 - xy)(1 - yz)(1 - xz)) = \\ &= (1 - xy)(1 - yz)(1 - xz) = (1 - xy - yz + xy^2 z)(1 - xz) = \\ &= (1 - xy - yz + xyz)(1 - xz) = 1 - xy - yz + xyz - xz + x^2 yz + xyz^2 - x^2 yz^2 = \\ &= 1 - xy - yz + xyz - xz + xyz + xyz - xyz = 1 - xy - yz - xz + 2xyz \end{aligned}$$

$$e) \quad \begin{aligned} \overline{x \oplus y \overline{xy}} &= (xy \vee \bar{x} \bar{y}) \overline{xy} = (1 - (1 - xy)(1 - (1 - x)(1 - y)))(1 - xy) = \\ &= (1 - (1 - xy)(x + y - xy))(1 - xy) = (1 - x - y + xy + x^2 y + xy^2 - x^2 y^2)(1 - xy) = \\ &= (1 - x - y + xy + xy + xy - xy)(1 - xy) = (1 - x - y + 2xy)(1 - xy) = \\ &= 1 - x - y + 2xy - xy + x^2 y + xy^2 - 2x^2 y^2 = 1 - x - y + 2xy - xy + xy + xy - 2xy = \\ &= 1 - x - y + xy \end{aligned}$$

Do istog rezultata se može doći brže i jednostavnije ukoliko se na početku prvo pojednostavi polazni izraz:

$$\begin{aligned} \overline{x \oplus y \overline{xy}} &= (xy \vee \bar{x} \bar{y}) \overline{xy} = (xy \vee \bar{x} \bar{y})(\bar{x} \vee \bar{y}) = \bar{x} \bar{y} \vee \bar{x} \bar{y} = \bar{x} \bar{y} = \\ &= (1 - x)(1 - y) = 1 - x - y + xy \end{aligned}$$

$$f) \quad \begin{aligned} (xy \vee z)(\bar{z} \vee xz) &= (1 - (1 - xy)(1 - z))(1 - z(1 - xz)) = \\ &= (1 - 1 + z + xy - xyz)(1 - z + xz^2) = (z + xy - xyz)(1 - z + xz) = \\ &= z - z^2 + xz^2 + xy - xyz + x^2 yz - xyz + xy z^2 - x^2 yz^2 = \\ &= z - z + xz + xy - xyz + xyz - xyz + xyz - xyz = xy + xz - xyz \end{aligned}$$

Alternativno, postupak se može skratiti uz prethodno pojednostavljivanje polaznog izraza:

$$\begin{aligned} (xy \vee z)(\bar{z} \vee xz) &= (xy \vee z)(x \vee \bar{z}) = xy \vee xy \bar{z} \vee xz = xy \vee xz = \\ &= x(y \vee z) = x(1 - (1 - y)(1 - z)) = x(y + z - yz) = xy + xz - xyz \end{aligned}$$

$$\begin{aligned}
\text{g)} \quad \overline{\bar{x}y \vee x\bar{z} \vee \bar{y}z} &= 1 - (1 - (1 - x)y)(1 - (1 - x(1 - z))(1 - (1 - y)z)) = \\
&= 1 - (1 - y + xy)(1 - (1 - x + xz)(1 - z + yz)) = \\
&= 1 - (1 - y + xy)(1 - 1 + z - yz + x - xz + xyz - xz + xz^2 - xyz^2) = \\
&= 1 - (1 - y + xy)(z - yz + x - xz + xyz - xz + xz - xyz) = 1 - (1 - y + xy)(x + z - xz - yz) = \\
&= 1 - x - z + xz + yz + xy + yz - xyz - y^2z - x^2y - xyz + x^2yz + xy^2z = \\
&= 1 - x - z + xz + yz + xy + yz - xyz - yz - xy - xyz + xyz + xyz = 1 - x - z + xz + yz
\end{aligned}$$

Alternativno, mogli smo na početku prvo pojednostaviti izraz koliko je god to moguće, čime bismo imali više predradnji, ali bi kasniji postupak bio kraći i jednostavniji:

$$\begin{aligned}
\overline{\bar{x}y \vee x\bar{z} \vee \bar{y}z} &= \overline{\bar{x}y \vee x\bar{z}} \vee \overline{\bar{y}z} = \overline{\bar{x}y \vee (x\bar{z})} \vee (\bar{y}z) = \overline{\bar{x}y \vee \bar{x}y \vee x\bar{z} \vee yz} = \\
&= \overline{\bar{x}y \vee x\bar{z}} \vee \bar{y}z = \overline{\bar{x}y(z \vee \bar{z})} \vee \bar{y}z = \overline{\bar{x}yz \vee \bar{x}y\bar{z}} \vee \bar{y}z = \\
&= (\overline{\bar{x}yz \vee yz}) \vee (\overline{\bar{x}y\bar{z} \vee x\bar{z}}) = yz \vee \bar{x}\bar{z} = 1 - (1 - yz)(1 - (1 - x)(1 - z)) = \\
&= 1 - (1 - yz)(x + z - xz) = 1 - x - z + xz + xyz + yz^2 - xyz^2 = \\
&= 1 - x - z + xz + xyz + yz - xyz = 1 - x - z + xz + yz
\end{aligned}$$

$$\begin{aligned}
\text{h)} \quad (x \vee y \vee z)(x \vee \bar{y} \vee z) &= (1 - (1 - x)(1 - y)(1 - z))(1 - (1 - x)y(1 - z)) = \\
&= (1 - (1 - x - y + xy)(1 - z))(1 - (y - xy)(1 - z)) = \\
&= (1 - 1 + x + y - xy + z - xz - yz + xyz)(1 - y + yz + xy - xyz) = \\
&= (x + y + z - xy - xz - yz + xyz)(1 - y + xy + yz - xyz) = \\
&= x - xy + x^2y + xyz - x^2yz + y - y^2 + xy^2 + y^2z - xy^2z + z - yz + xyz + yz^2 - \\
&\quad - xy^2z^2 - xy + xy^2 - x^2y^2 - xy^2z + x^2y^2z - xz + xyz - x^2yz - xy^2z^2 + x^2y^2z^2 - \\
&\quad - yz + y^2z - xy^2z - y^2z^2 + xy^2z^2 + xyz - xy^2z + x^2y^2z + xy^2z^2 - x^2y^2z^2 = \\
&= x - xy + xy + xyz - xyz + y - y + xy + yz - xyz + z - yz + xyz + yz - xyz - xy + xy - xy - xyz + \\
&\quad + xyz - xz + xyz - xyz - xyz + xyz - yz + yz - xyz - yz + xyz + xyz - xyz + xyz + xyz - xyz = \\
&= x + z - xz
\end{aligned}$$

Postupak se može drastično pojednostaviti i skratiti ukoliko se prethodno pojednostavi polazni izraz, kao što je urađeno u sljedećem postupku:

$$\begin{aligned}
(x \vee y \vee z)(x \vee \bar{y} \vee z) &= ((x \vee z) \vee y)((x \vee z) \vee \bar{y}) = x \vee z \vee y \bar{y} = x \vee z \vee 0 = x \vee z = \\
&= 1 - (1 - x)(1 - z) = x + z - xz
\end{aligned}$$

$$\begin{aligned}
\text{i)} \quad \overline{\overline{xy \vee \bar{z}} \vee \overline{xy \vee \bar{z}}} &= 1 - (1 - (1 - xy)z)(1 - (1 - x)yz) = \\
&= 1 - (1 - z + xyz)(1 - yz + xyz) = 1 - 1 + yz - xyz + z - yz^2 + xyz^2 - xyz + xy^2z^2 - x^2y^2z^2 = \\
&= yz - xyz + z - yz + xyz - xyz + xyz - xyz = z - xyz
\end{aligned}$$

I u ovom slučaju ukoliko se prethodno izvrši pojednostavljenje polaznog izraza moguće je osjetno skratiti kasniji tok postupka:

$$\begin{aligned}
\overline{\overline{xy \vee \bar{z}} \vee \overline{xy \vee \bar{z}}} &= \overline{\bar{y}z \vee \bar{x}yz} = (\bar{x} \vee \bar{y})z \vee \bar{x}yz = \bar{x}z \vee \bar{y}z \vee \bar{x}yz = \bar{x}z \vee \bar{y}z = \\
&= (\bar{x} \vee \bar{y})z = (1 - xy)z = z - xyz
\end{aligned}$$