

Svedimo prvo izraz na oblik DNF:

$$\begin{aligned} B(A \Leftrightarrow C) \vee D(A \underline{\vee} C) &= B(AC \vee \bar{A}\bar{C}) \vee D(\bar{A}C \vee A\bar{C}) = \\ &= ABC \vee \bar{A}\bar{B}\bar{C} \vee \bar{A}C \vee A\bar{C} \end{aligned}$$

Kako DNF nije jedinstvena, ovo je samo jedan od mogućih oblika DNF (može se pokazati da je ovo najkraća DNF polaznog izraza, vidi Zadatak 1.42). Proširimo dobijeni izraz do oblika SDNF:

$$\begin{aligned} B(A \Leftrightarrow C) \vee D(A \underline{\vee} C) &= ABC \vee \bar{A}\bar{B}\bar{C} \vee \bar{A}C \vee A\bar{C} = \\ &= ABC(D \vee \bar{D}) \vee \bar{A}\bar{B}\bar{C}(D \vee \bar{D}) \vee \bar{A}C(D \vee \bar{D}) \vee A\bar{C}(D \vee \bar{D}) = \\ &= ABCD \vee ABC\bar{D} \vee \bar{A}\bar{B}\bar{C}D \vee \bar{A}\bar{B}\bar{C}\bar{D} \vee \bar{A}CD \vee \bar{A}\bar{C}D \vee \bar{A}C\bar{D} \vee \bar{A}\bar{C}\bar{D} \end{aligned}$$

Kako je SDNF jedinstvena do na poredak članova, svi oblici SDNF polaznog izraza dobijeni na neki drugi način mogu se razlikovati od ovog samo u poretku članova.

Nađimo sada oblik KNF zadanog izraza:

$$\begin{aligned} B(A \Leftrightarrow C) \vee D(A \underline{\vee} C) &= B(AC \vee \bar{A}\bar{C}) \vee D(\bar{A}C \vee A\bar{C}) = \\ &= B(A \vee \bar{A}\bar{C})(C \vee \bar{A}\bar{C}) \vee D(A \vee \bar{A}C)(\bar{C} \vee \bar{A}C) = B(A \vee \bar{C})(\bar{A} \vee C) \vee D(A \vee C)(\bar{A} \vee \bar{C}) = \\ &= (B(A \vee \bar{C})(\bar{A} \vee C) \vee D)(B(A \vee \bar{C})(\bar{A} \vee C) \vee (A \vee C))(B(A \vee \bar{C})(\bar{A} \vee C) \vee (\bar{A} \vee \bar{C})) = \\ &= (B \vee D)(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D)(B \vee A \vee C)(A \vee \bar{C} \vee A \vee C)(\bar{A} \vee C \vee A \vee C) \dots \\ &\quad \dots (B \vee \bar{A} \vee \bar{C})(A \vee \bar{C} \vee \bar{A} \vee \bar{C})(\bar{A} \vee C \vee \bar{A} \vee \bar{C}) = \\ &= (B \vee D)(A \vee B \vee C)(\bar{A} \vee B \vee \bar{C})(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D) \end{aligned}$$

Razumije se da je ovo samo jedan od mogućih oblika KNF, pošto ni KNF nije jedinstvena. Na primjer, elementarnim manipulacijama moguće je dobiti kraći oblik KNF za isti izraz, jer se prvog faktora $B \vee D$ možemo osloboditi:

$$\begin{aligned} (B \vee D)(A \vee B \vee C)(\bar{A} \vee B \vee \bar{C})(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D) &= \\ &= (B \vee D \vee A\bar{A} \vee C\bar{C})(A \vee B \vee C)(\bar{A} \vee B \vee \bar{C})(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D) = \\ &= (B \vee C \vee D \vee A\bar{A})(B \vee \bar{C} \vee D \vee A\bar{A})(A \vee B \vee C)(\bar{A} \vee B \vee \bar{C})(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D) = \\ &= (A \vee B \vee C \vee D)(\bar{A} \vee B \vee C \vee D)(A \vee B \vee \bar{C} \vee D)(\bar{A} \vee B \vee \bar{C} \vee D) \dots \\ &\quad \dots (A \vee B \vee C)(\bar{A} \vee B \vee \bar{C})(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D) = \\ &= (A \vee B \vee C)(\bar{A} \vee B \vee \bar{C})(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D) \end{aligned}$$

Ovdje smo koristili činjenicu da je $X(X \vee Y) = X$. Može se pokazati da je ova KNF polaznog izraza najkraća moguća (vidi Zadatak 1.42). Nađimo sada oblik SKNF. Za tu svrhu, možemo krenuti od bilo kojeg oblika KNF, tako da ćemo krenuti od onog oblika koji smo prvo pronašli:

$$\begin{aligned} (B \vee D)(A \vee B \vee C)(\bar{A} \vee B \vee \bar{C})(A \vee \bar{C} \vee D)(\bar{A} \vee C \vee D) &= \\ &= (B \vee D \vee A\bar{A} \vee C\bar{C})(A \vee B \vee C \vee D\bar{D})(\bar{A} \vee B \vee \bar{C} \vee D\bar{D})(A \vee \bar{C} \vee D \vee B\bar{B})(\bar{A} \vee C \vee D \vee B\bar{B}) = \\ &= (A \vee B \vee D \vee C\bar{C})(\bar{A} \vee B \vee D \vee C\bar{C})(A \vee B \vee C \vee D\bar{D}) \dots \\ &\quad \dots (\bar{A} \vee B \vee \bar{C} \vee D\bar{D})(A \vee \bar{C} \vee D \vee B\bar{B})(\bar{A} \vee C \vee D \vee B\bar{B}) = \\ &= (A \vee B \vee C \vee D)(A \vee B \vee \bar{C} \vee D)(\bar{A} \vee B \vee C \vee D)(\bar{A} \vee B \vee \bar{C} \vee D) \dots \\ &\quad \dots (A \vee B \vee C \vee D)(A \vee B \vee C \vee \bar{D})(\bar{A} \vee B \vee \bar{C} \vee D)(\bar{A} \vee B \vee \bar{C} \vee \bar{D}) \dots \\ &\quad \dots (A \vee B \vee \bar{C} \vee D)(A \vee \bar{B} \vee \bar{C} \vee D)(\bar{A} \vee B \vee C \vee D)(\bar{A} \vee \bar{B} \vee C \vee D) = \\ &= (A \vee B \vee C \vee D)(A \vee B \vee \bar{C} \vee D)(\bar{A} \vee B \vee C \vee D)(\bar{A} \vee B \vee \bar{C} \vee D) \dots \\ &\quad \dots (A \vee B \vee C \vee \bar{D})(\bar{A} \vee B \vee \bar{C} \vee \bar{D})(A \vee \bar{B} \vee \bar{C} \vee D)(\bar{A} \vee \bar{B} \vee C \vee D) \end{aligned}$$

Kako je SKNF također jedinstvena do na poredak članova, svi oblici SDNF polaznog izraza dobijeni na neki drugi način mogu se razlikovati od ovog samo u poretku članova.

Izvedimo još oblike KNF i SKNF polaznog izraza metodom dvojne negacije. Svedimo prvo negaciju polaznog izraza na oblik DNF:

$$\begin{aligned} \overline{B(A \Leftrightarrow C) \vee D(A \underline{\vee} C)} &= \overline{B(AC \vee \overline{A} \overline{C}) \vee D(\overline{A} \overline{C} \vee \overline{A} C)} = \overline{B(AC \vee \overline{A} \overline{C})} \overline{D(\overline{A} \overline{C} \vee \overline{A} C)} = \\ &= (\overline{B} \vee \overline{AC \vee \overline{A} \overline{C}}) (\overline{D} \vee \overline{\overline{A} \overline{C} \vee \overline{A} C}) = (\overline{B} \vee \overline{AC} \overline{A} \overline{C}) (\overline{D} \vee \overline{A} \overline{C} \overline{A} C) = \\ &= (\overline{B} \vee (\overline{A} \vee \overline{C})(A \vee C)) (\overline{D} \vee (\overline{A} \vee C)(A \vee \overline{C})) = (\overline{B} \vee \overline{AC} \vee \overline{AC}) (\overline{D} \vee \overline{A} \overline{C} \vee AC) = \\ &= \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D \vee A \overline{A} C \vee A \overline{C} \overline{D} \vee A \overline{C} C \vee A C \overline{D} = \\ &= \overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D \end{aligned}$$

Negiranjem ovog izraza dobijamo KNF polaznog izraza:

$$\begin{aligned} B(A \Leftrightarrow C) \vee D(A \underline{\vee} C) &= \overline{\overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D} = \overline{\overline{B} \overline{D}} \overline{\overline{A} \overline{B} \overline{C}} \overline{\overline{A} \overline{B} C} \overline{\overline{A} C \overline{D}} \overline{\overline{A} C D} = \\ &= (B \vee D)(A \vee B \vee C)(\overline{A} \vee B \vee \overline{C})(A \vee \overline{C} \vee D)(\overline{A} \vee C \vee D) \end{aligned}$$

Skraćeni oblik KNF koji smo ranije izveli možemo dobiti ukoliko prvo korištenjem elementarnih transformacija skratimo dobijeni oblik DNF negacije polaznog izraza:

$$\begin{aligned} \overline{\overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D} &= \overline{\overline{B} \overline{D}} (\overline{A} \vee \overline{A}) (C \vee \overline{C}) \vee \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D = \\ &= \overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} D \vee \overline{A} \overline{B} C \overline{D} \vee \overline{A} \overline{B} C D \vee \overline{A} B \overline{C} \vee \overline{A} B C \vee \overline{A} C \overline{D} \vee \overline{A} C D = \\ &= \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D \end{aligned}$$

Ovdje smo iskoristili pravilo sažimanja $X \vee XY = X$ gdje god je to bilo moguće. Sada negacijom dobijenog DNF oblika dobijamo traženi skraćeni oblik KNF polaznog izraza:

$$\begin{aligned} \overline{\overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D} &= \overline{\overline{A} \overline{B} \overline{C}} \overline{\overline{A} \overline{B} C} \overline{\overline{A} C \overline{D}} \overline{\overline{A} C D} = \\ &= (A \vee B \vee C)(\overline{A} \vee B \vee \overline{C})(A \vee \overline{C} \vee D)(\overline{A} \vee C \vee D) \end{aligned}$$

Za nalaženje SKNF polaznog izraza metodom dvojne negacije, proširimo neki od nađenih oblika DNF negacije polaznog izraza do oblika SDNF, recimo onaj oblik koji smo prvi našli:

$$\begin{aligned} \overline{\overline{B} \overline{D} \vee \overline{A} \overline{B} \overline{C} \vee \overline{A} \overline{B} C \vee \overline{A} C \overline{D} \vee \overline{A} C D} &= \\ &= \overline{\overline{B} \overline{D}} (\overline{A} \vee \overline{A}) (C \vee \overline{C}) \vee \overline{A} \overline{B} \overline{C} (D \vee \overline{D}) \vee \overline{A} \overline{B} C (D \vee \overline{D}) \vee \overline{A} C \overline{D} (B \vee \overline{B}) \vee \overline{A} C D (B \vee \overline{B}) = \\ &= \overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} D \vee \overline{A} \overline{B} C \overline{D} \vee \overline{A} \overline{B} C D \vee \overline{A} B \overline{C} \overline{D} \vee \overline{A} B \overline{C} D \vee \overline{A} B C \overline{D} \vee \overline{A} B C D = \\ &= \overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} D \vee \overline{A} \overline{B} C \overline{D} \vee \overline{A} \overline{B} C D \vee \overline{A} B \overline{C} \overline{D} \vee \overline{A} B \overline{C} D \vee \overline{A} B C \overline{D} \vee \overline{A} B C D \end{aligned}$$

Sada, negacijom ovog izraza dobijamo oblik SKNF polaznog izraza:

$$\begin{aligned} \overline{\overline{A} \overline{B} \overline{C} \overline{D} \vee \overline{A} \overline{B} \overline{C} D \vee \overline{A} \overline{B} C \overline{D} \vee \overline{A} \overline{B} C D \vee \overline{A} B \overline{C} \overline{D} \vee \overline{A} B \overline{C} D \vee \overline{A} B C \overline{D} \vee \overline{A} B C D} &= \\ &= \overline{\overline{A} \overline{B} \overline{C} \overline{D}} \overline{\overline{A} \overline{B} \overline{C} D} \overline{\overline{A} \overline{B} C \overline{D}} \overline{\overline{A} \overline{B} C D} \overline{\overline{A} B \overline{C} \overline{D}} \overline{\overline{A} B \overline{C} D} \overline{\overline{A} B C \overline{D}} \overline{\overline{A} B C D} = \\ &= (\overline{A} \vee B \vee \overline{C} \vee D)(\overline{A} \vee B \vee C \vee D)(A \vee B \vee \overline{C} \vee D)(A \vee B \vee C \vee D) \dots \\ &\dots (A \vee B \vee C \vee \overline{D})(\overline{A} \vee B \vee \overline{C} \vee D)(A \vee \overline{B} \vee \overline{C} \vee D)(\overline{A} \vee \overline{B} \vee C \vee D) \end{aligned}$$

Dobili smo oblik koji se od oblika SKNF dobijenog direktnim putem razlikuje samo u poretku članova.