

$$\begin{aligned}
\text{a)} \quad & \overline{(A \vee \overline{BCD})(\overline{AD} \vee B(\overline{C} \vee A))} = \overline{A \vee \overline{BCD} \vee \overline{AD} \vee B(\overline{C} \vee A)} = \overline{A \overline{BCD} \vee \overline{AD} B(\overline{C} \vee A)} = \\
& = \overline{A(\overline{B} \vee \overline{C} \vee \overline{D}) \vee (\overline{A} \vee \overline{D})(\overline{B} \vee \overline{C} \vee A)} = \overline{A(B \vee C \vee D) \vee (A \vee D)(\overline{B} \vee \overline{C} \vee A)} = \\
& = \overline{A(B \vee C \vee D) \vee (A \vee D)(\overline{B} \vee \overline{AC})} = \overline{A(B \vee C \vee D) \vee (\overline{AB} \vee A\overline{AC} \vee \overline{BD} \vee \overline{ACD})} = \\
& = \overline{AB \vee (\overline{AC} \vee \overline{ACD}) \vee \overline{AD} \vee \overline{AB} \vee \overline{BD}} = \overline{AB \vee \overline{AC} \vee \overline{AD} \vee \overline{AB} \vee \overline{BD}} = \\
& = \overline{AB \vee \overline{AC} \vee \overline{AD} \vee \overline{AB} \vee \overline{BD}(A \vee \overline{A})} = \overline{AB \vee \overline{AC} \vee \overline{AD} \vee \overline{AB} \vee \overline{ABD} \vee \overline{ABD}} = \\
& = \overline{AB \vee \overline{AC} \vee (\overline{AD} \vee \overline{ABD}) \vee (\overline{AB} \vee \overline{ABD})} = \overline{AB \vee \overline{AC} \vee \overline{AD} \vee \overline{AB}} = \\
& = \overline{(\overline{AB} \vee \overline{AB}) \vee \overline{A}(C \vee D)} = (A \vee B) \vee \overline{A}(C \vee D)
\end{aligned}$$

Teži dio ovog zadatka je eliminacija člana \overline{BD} , što je učinjeno na sličan način kao u prvom primjeru na stranici 22 u udžbeniku. Također, da nismo željeli koristiti ekskluzivnu disjunkciju, najkraći izraz koji bismo mogli dobiti glasi $\overline{A}(B \vee C \vee D) \vee \overline{AB}$.

$$\begin{aligned}
\text{b)} \quad & \overline{ABC \vee (\overline{A} \vee B \vee D)(ABD \vee \overline{B})} = \overline{ABC(\overline{A} \vee B \vee D)(ABD \vee \overline{B})} = \\
& = \overline{(\overline{A} \vee \overline{B} \vee \overline{C})(\overline{A} \vee B \vee D \vee \overline{ABD} \vee \overline{B})} = \overline{(A \vee \overline{B} \vee \overline{C})(\overline{ABD} \vee \overline{ABD} \vee \overline{B})} = \\
& = \overline{(A \vee \overline{B} \vee \overline{C})(\overline{ABD} \vee (\overline{A} \vee \overline{B} \vee D)B)} = \overline{(A \vee \overline{B} \vee \overline{C})(\overline{ABD} \vee \overline{AB} \vee \overline{BB} \vee \overline{BD})} = \\
& = \overline{(A \vee \overline{B} \vee \overline{C})(\overline{ABD} \vee \overline{AB} \vee \overline{BD})} = \\
& = \overline{ABD \vee A\overline{AB} \vee ABD \vee \overline{ABD} \vee \overline{AB} \vee \overline{BBD} \vee \overline{ABCD} \vee \overline{ABC} \vee \overline{BCD}} = \\
& = \overline{ABD \vee ABD \vee \overline{ABCD} \vee \overline{ABC} \vee \overline{BCD}} = \overline{ABD \vee ABD \vee (\overline{ABC} \vee \overline{ABCD}) \vee \overline{BCD}} = \\
& = \overline{ABD \vee ABD \vee \overline{ABC} \vee \overline{BCD}} = \overline{ABD \vee ABD \vee \overline{ABC} \vee \overline{BCD}(A \vee \overline{A})} = \\
& = \overline{ABD \vee ABD \vee \overline{ABC} \vee \overline{ABCD} \vee \overline{ABC} \vee \overline{BCD}} = \overline{ABD \vee (ABD \vee \overline{ABCD}) \vee (\overline{ABC} \vee \overline{ABCD})} = \\
& = \overline{ABD \vee ABD \vee \overline{ABC}} = A(BD \vee \overline{BD}) \vee \overline{ABC} = A(B \Leftrightarrow D) \vee \overline{ABC}
\end{aligned}$$

Do oblika $\overline{ABD} \vee ABD \vee \overline{ABC} \vee \overline{BCD}$ se sasvim lako dolazi. Interesantniji i teži dio ovog zadatka je eliminacija člana \overline{BCD} , što je također učinjeno na sličan način kao u prvom primjeru na stranici 22 u udžbeniku.

Napomena: Da smo slučajno umjesto izraza $\overline{ABD} \vee \overline{ABD} \vee \overline{ABC} \vee \overline{BCD}$ dobili izraz u kojem je zadnja pojava promjenljive D negirana (tj. izraz $\overline{ABD} \vee \overline{ABD} \vee \overline{ABC} \vee \overline{BCD}$), dalji postupak pojednostavljivanja bi bio znatno manje očigledan. S obzirom da se u tom postupku koriste neke interesantne transformacije koje zaista nisu očigledne na prvi pogled, ovdje će radi ilustracije nekih korisnih ideja biti prikazane i taj postupak:

$$\begin{aligned}
& \overline{ABD} \vee \overline{ABD} \vee \overline{ABC} \vee \overline{BCD} = \overline{ABD} \vee (\overline{ABD} \vee \overline{ABCD}) \vee \overline{ABC} \vee (\overline{BCD} \vee \overline{ABCD}) = \\
& = \overline{ABD} \vee \overline{ABD} \vee (\overline{ABCD} \vee \overline{ABCD}) \vee \overline{ABC} \vee \overline{BCD} = \\
& = \overline{ABD} \vee \overline{ABD} \vee (\overline{ABC} \vee \overline{ABC}) \vee \overline{BCD} = \overline{ABD} \vee \overline{ABD} \vee (\overline{BC} \vee \overline{BCD}) = \\
& = \overline{ABD} \vee \overline{ABD} \vee \overline{BC} = A(BD \vee \overline{BD}) \vee \overline{BC} = A(B \Leftrightarrow D) \vee \overline{BC}
\end{aligned}$$

Naravno, Quineov algoritam nas oslobađa potrebe za pronalaženjem ovakvih neočiglednih “ad hoc” transformacija i omogućava da se postupak minimizacije provede rutinski.